# Decision rules

**Key concepts & study plan**

Experimental design

Data collection & processing

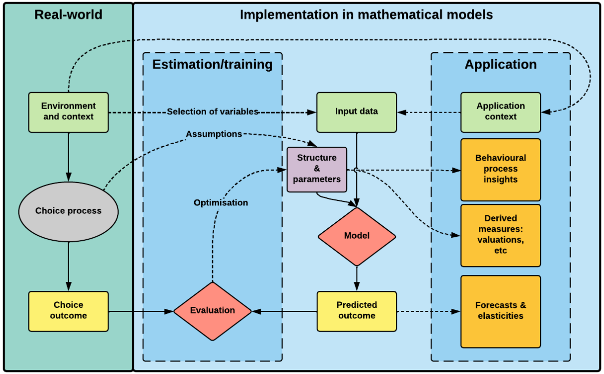
Model specification & estimation

Interpretation & application

# Decision rules

## The whole story

- Inidividuals make choices
- Analyst represents choices in a model
  1. Define a structure
  2. Estimate parameters
  3. Apply model and produce outputs



# Role of the decision rule

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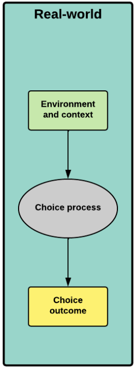
# Role of the decision rule

## Real-world process

- Decision-maker:
  - faces choice situation
  - uses an internal process
  - reaches outcome
- Analyst:
  - observes inputs (maybe in part)
  - observes outcome
  - does NOT observe process



# Role of the decision rule

## From real-world process to mathematical model

**Step 1: observe choice**

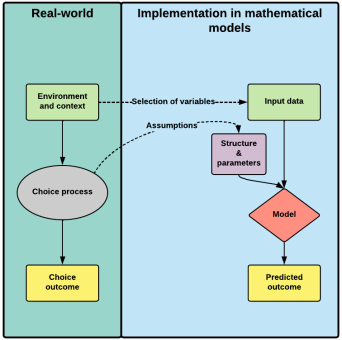
- dependent variable for our model ($Y$)

**Step 2: identify factors influencing choices**

- characteristics of alternatives ($x$), choice setting ($w$), and decision-maker ($z$)

**Step 3: Build model**

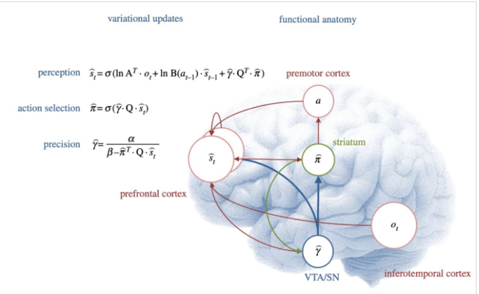
- $Y = m(\beta, x, w, z)$, where $m()$ reflects model structure and $\beta$ are parameters



# Role of the decision rule

## Decision rules

- Mathematical representation of choice process
- Does not imply that people make choices according to the rules we use
- Simply *convenient* way of representing process
- Factors to consider
  - behavioural realism
  - tractability
  - properties of outputs



# Compensatory *vs* non-compensatory

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# Compensatory *vs* non-compensatory

## Two contrasting approaches

### Compensatory models

- Changes in one attribute can be counteracted by changes in another attribute
- Theory: increases in cost can always be counteracted by reductions in time
- Practice: depends on size of change that is needed
- Key foundation of random utility maximisation (RUM)

### Non-compensatory and semi-non-compensatory models

- Some changes cannot be counteracted, or only partially
- Non-compensatory example: Elimination by aspects (EBA)
- Semi-non-compensatory example: Random Regret Minimisation (RRM)

# Compensatory *vs* non-compensatory

## Elimination by aspects (EBA): overview

- Gradual elimination of alternatives

| Attribute | Drug A | Drug B | Drug C | Drug D | Rule |
|---|---|---|---|---|---|
| Cost | \$5 per day | \$10 per day | \$15 per day | \$20 per day | Cost $\leqslant$ \$15 |
| Risk | 1 in 500 | 1 in 1,000 | 1 in 2,000 | 1 in 5,000 | Risks $\leqslant$1 in 1,000 |
| Success | 60% | 70% | 80% | 90% | Success rate $\geqslant$ 75% |

- Clearly not compensatory

Key reference: *Tversky, A. (1972), 'Elimination by aspects: A theory of choice', Psychological Review 79, 281-299.*

# Utility theory

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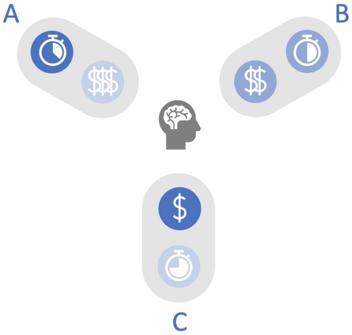
# Utility theory

## General concept of utility maximisation

- Notion grounded in micro-economic theory
- Alternatives characterised by utility
  - based on attributes which influence behaviour
- Assume rational behaviour
  - choose alternative with highest utility
- Trade-off behaviour:
  - good performance on one attribute compensates for poor performance on another
  - e.g. higher cost compensated by faster journey




Key reference: *Marschak, J. (1960), 'Binary choice constraints on random utility indications', in K. Arrow, ed., Stanford Symposium on Mathematical Methods in the Social Sciences, Stanford University Press, Stanford, CA, pp. 312-329.*

# Utility theory

## Utility specification

### Decision-maker

- Person $n$, with $n = 1, \ldots, N$
- Faces $T_n$ choice situations, with $t = 1, \ldots, T_n$
- Characteristics $z_n$ (observed)
- Vector of preferences/tastes $\beta_n$ (estimated)

### Choice-set and context

- $J$ mutually exclusive alternatives, with $j = 1, \ldots, J$
- Choice context described by $w_{nt}$
- Alt. $j$ described by set of $K$ attributes
- In situation $t$, $x_{jnt} = \langle x_{jnt,1}, \ldots, x_{jnt,K} \rangle$

$$\boxed{U_{jnt} = f\left(\beta_n, x_{jnt}, w_{nt}, z_n\right)}$$

# Utility theory

## Utility and choice

- For now, drop indices $n$ and $t$
- Choice index given by $Y$
- Alternative with highest utility is chosen

$$\boxed{Y = i \iff U_i > U_j \;\; \forall j \neq i}$$

- Observation:
  **only differences in utility matter**

$$\boxed{\begin{array}{c} U_j^* = U_j + \Delta \;\; \forall j \\ Y^* = Y \;\; \forall \Delta \end{array}}$$

# Utility theory

## A simple example

- Choice between two train services
- Alternatives can only be distinguished via their attributes
- Both attributes are numerical
- Use a linear in attributes specification
- If $T_1$ increases by one minute, $U_1$ changes by $\beta_T$
- Expect $\beta_T$ and $\beta_C$ to be negative

### Unlabelled choice alternatives

| | Train 1 | Train 2 |
|---|---|---|
| Travel time (T) | 45 min | 30 min |
| Travel cost (C) | £7 | £12 |

### Utility specification

$$U_1 = \beta_T T_1 + \beta_C C_1$$

$$U_2 = \beta_T T_2 + \beta_C C_2$$

# Utility theory

## Choice outcome

- Choice depends on differences in utilities
- If sensitivity to time increases, differences in time matter more (same for cost)
- If all sensitivities increase by same factor, order of preferences does not change

**Utility specification**

$$U_1 = \beta_T T_1 + \beta_C C_1$$
$$U_2 = \beta_T T_2 + \beta_C C_2$$

**Choice outcome**

$$Y = 1 \iff \beta_T (T_1 - T_2) > \beta_C (C_2 - C_1)$$

$$\beta_T, \beta_C < 0 \Rightarrow \frac{\beta_T}{\beta_C} (T_2 - T_1) > C_1 - C_2$$

# Utility theory

## Example for our choice task: $T_1 > T_2$ and $C_1 < C_2$

- Option 1 will be chosen if $\frac{\beta_T}{\beta_C} < \frac{C_2 - C_1}{T_1 - T_2}$
  - not willing to pay extra cost to save time
- Option 2 will be chosen if $\frac{\beta_T}{\beta_C} > \frac{C_2 - C_1}{T_1 - T_2}$
- No information from observations with *dominant* alternative
- To find $\beta$ values, need many observations with changing attribute levels

### Choice scenario

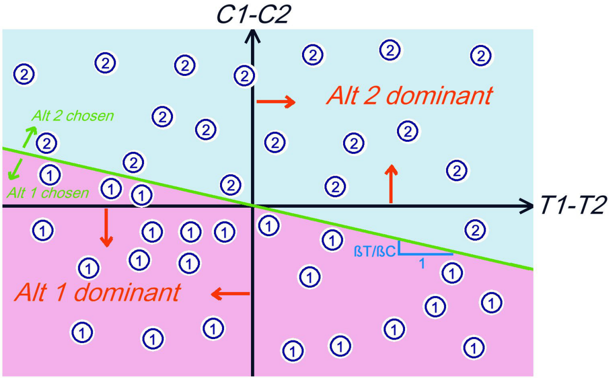
| | Train 1 | Train 2 |
|---|---|---|
| Travel time (T) | 45 min | 30 min |
| Travel cost (C) | £7 | £12 |

### Choice outcome

$$Y = 1 \iff \frac{\beta_T}{\beta_C} < \frac{C_2 - C_1}{T_1 - T_2}$$

# Utility theory

## We can solve this graphically



# Deterministic *vs* random utility theory

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# Deterministic *vs* random utility theory

## Shortcomings of deterministic utility theory

- Often make observations of
  - inconsistent behaviour
  - non-transitive preferences
- Cause of inconsistencies cannot be specified in deterministic framework
  - lack of analyst's knowledge of individual's decision processes
  - unobserved attributes
  - unobserved heterogeneity
  - incorrectly measured attributes
  - poor information on availabilities
  - non-linearities in preferences
- To accommodate this, we move to a probabilistic model

# Deterministic *vs* random utility theory

## Random utility theory

- Utility $U_{jn}$ is a random variable
  - *deterministic* part $V_{jn}$
  - *random* part $\varepsilon_{jn}$
- Deterministic part specified to capture role of observed explanators
- $\varepsilon_{jn}$ measures deviation from modelled utility for alternative $j$ and respondent $n$

**Additive utility structure**

$$U_{jn} = V_{jn} + \varepsilon_{jn}$$

**Deterministic part of utility**

$$V_{jn} = f\left(\beta_n, x_{jn}\right)$$

# Deterministic *vs* random utility theory

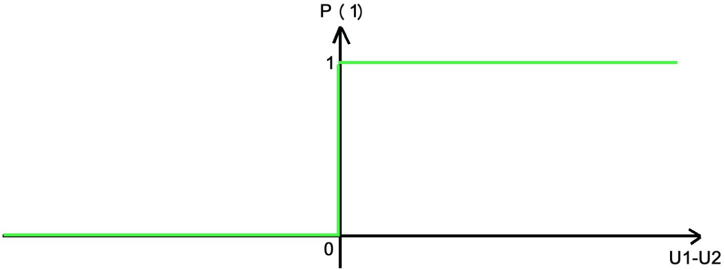
## Implications for probabilities

- Deterministic utility theory
  - Alternative with highest utility is chosen
- Random utility theory
  - Probability of choosing alternative increases with deterministic utility
- Probability of person $n$ choosing alternative $i$ given by:

$$\boxed{P_{in} = \Pr\left(V_{in} + \varepsilon_{in} > V_{jn} + \varepsilon_{jn} \quad \forall j \neq i\right)}$$

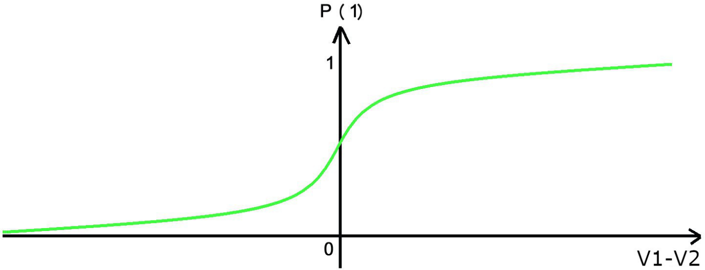
# Deterministic *vs* random utility theory

## Binary deterministic choice



# Deterministic *vs* random utility theory

## Binary probabilistic choice



# Deterministic *vs* random utility theory

## Only differences in utility matter

- Probability of person $n$ choosing alternative $i$ given by:

$$\boxed{P_{in} = \Pr\left(V_{in} + \varepsilon_{in} > V_{jn} + \varepsilon_{jn} \quad \forall j \neq i\right)}$$

- Adding same value to $V_{jn} \quad \forall j$ will not change probabilities
- Observation: <u>only differences in utilities matter</u>
- Implication: <u>parameters only identified if they capture differences across alternatives</u>
  - require normalisation for e.g. alternative specific constants