Working in willingness-to-pay (WTP) space

Key concepts & study plan

Experimental design

Data collection & processing

Model specification & estimation

Interpretation & application

WTP space: introduction

- Can reparameterise model in WTP space as opposed to utility/preference space
- By rescaling marginal utilities of (some) non-cost attributes by cost coefficient
- WTP now directly estimated
  - rather than requiring calculation as ratio of partial derivatives
  - also directly obtain standard errors
- Not a different model, simply a reparameterisation of utility function
- For fixed parameter models, the two specifications are mathematically equivalent

Theory

Understanding reparameterisation of utilities

- Simple example where T is time in minutes, and C is cost in £

Specification 1

$$V_{ni} = \beta_T\, T_{ni} + \beta_C\, C_{ni} + \ldots$$

Marginal utilities wrt one minute in T and £1 in C given by β_T and β_C

Specification 2

$$V_{ni} = \frac{1}{60}\beta'_T T_{ni} + 100\beta'_C C_{ni} + \ldots$$

Marginal utilities wrt one min in T and £1 in C given by $\frac{1}{60}\beta'_T$ and $100\beta'_C$

- Models mathematically equivalent, so we have that $\beta'_T = 60\beta_T$ (i.e., expressed in hours) and $\beta'_C = \frac{1}{100}\beta_C$ (i.e., expressed in pence)
- Can use same rationale to reparameterise actual numeraire

Theory

WTP space: implementation

- Rescale marginal utilities of (some) non-cost attributes by cost coefficient

Utility space

$$V_{ni} = \beta_T\, T_{ni} + \beta_C\, C_{ni} + \ldots$$
$$\frac{\partial V_{ni}}{\partial T_{ni}} = \beta_T \;\&\; \frac{\partial V_{ni}}{\partial C_{ni}} = \beta_C$$
$$\frac{\partial V_{ni}}{\partial T_{ni}} \Big/ \frac{\partial V_{ni}}{\partial C_{ni}} = \frac{\beta_T}{\beta_C}$$

WTP space

$$V_{ni} = \beta'_C\, \left(\beta'_{VT}\, T_{ni} + C_{ni}\right) + \ldots$$
$$\frac{\partial V_{ni}}{\partial T_{ni}} = \beta'_C\, \beta'_{VT} \;\&\; \frac{\partial V_{ni}}{\partial C_{ni}} = \beta'_C$$
$$\frac{\partial V_{ni}}{\partial T_{ni}} \Big/ \frac{\partial V_{ni}}{\partial C_{ni}} = \beta'_{VT}$$

- With fixed coefficients, this is a simple rescaling, and the two models are thus mathematically equivalent, with $\beta_T = \beta'_C\, \beta'_{VT}$, $\beta_C = \beta'_C$, and thus $\frac{\beta_T}{\beta_C} = \beta'_{VT}$

Key reference: *Train, K. & Weeks, M. (2006). Discrete Choice Models in Preference Space and Willingness-to Pay Space. In: Alberini, A. & Scarpa, R. (eds) Applications of Simulation Methods in Environmental and Resource Economics, Springer*

Illustration on mode choice data

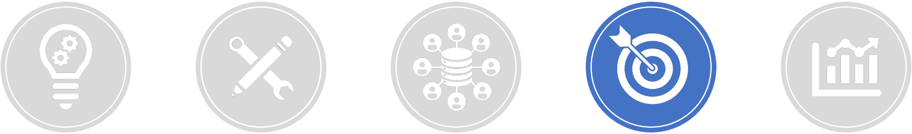

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Illustration on mode choice data

Models in preference space and WTP space

- Mathematically equivalent

```
LL(start)                             : -8196.02
LL at equal shares, LL(0)             : -8196.02
LL at observed shares, LL(C)          : -6706.94
LL(final)                             : -5615.39
Rho-squared vs equal shares            :  0.3149
Adj.Rho-squared vs equal shares        :  0.3139
Rho-squared vs observed shares         :  0.1627
Adj.Rho-squared vs observed shares     :  0.162
AIC                                   :  11246.78
BIC                                   :  11301.61
```

```
Unconstrained optimisation.

Estimates:
               Estimate        s.e.   t.rat.(0)    Rob.s.e. Rob.t.rat.(0)
asc_car         0.00000          NA          NA          NA            NA
asc_bus        -2.04288    0.075132     -27.191    0.092220       -22.152
asc_air        -0.58780    0.180223      -3.262    0.197274        -2.980
asc_rail       -0.86198    0.107216      -8.040    0.117824        -7.316
b_tt           -0.01205  5.5356e-04     -21.775  5.9548e-04       -20.242
b_access       -0.01992    0.002507      -7.946    0.002489        -8.003
b_cost         -0.05870    0.001463     -40.118    0.001680       -34.951
b_no_frills     0.00000          NA          NA          NA            NA
b_wifi          0.95150    0.052893      17.989    0.055165        17.248
b_food          0.41168    0.052141       7.895    0.052807         7.796
```

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asc_air        -0.58780    0.180223      -3.262    0.197274        -2.980
asc_rail       -0.86198    0.107216      -8.040    0.117824        -7.316
v_tt            0.20533    0.008783      23.379    0.009523        21.563
v_access        0.33933    0.042442       7.995    0.042270         8.028
b_cost         -0.05870    0.001463     -40.118    0.001680       -34.951
v_no_frills     0.00000          NA          NA          NA            NA
v_wifi        -16.20835    0.896314     -18.083    1.003308       -16.155
v_food         -7.01269    0.881983      -7.951    0.894946        -7.836
```

Illustration on mode choice data

Same findings for MRS, and same standard errors

- Understand the reason for the signs of MRS?

```
Unconstrained optimisation.

These outputs have had the scaling used in estimation applied to them.
Estimates:
               Estimate        s.e.   t.rat.(0)    Rob.s.e. Rob.t.rat.(0)
asc_car         0.00000          NA          NA          NA            NA
asc_bus        -2.04288    0.075131     -27.191    0.092220       -22.152
asc_air        -0.58781    0.180223      -3.262    0.197274        -2.980
asc_rail       -0.86199    0.107216      -8.040    0.117824        -7.316
b_tt           -0.01205  5.5356e-04     -21.775  5.9548e-04       -20.242
b_access       -0.01992    0.002507      -7.946    0.002489        -8.003
b_cost         -0.05870    0.001463     -40.118    0.001680       -34.951
b_no_frills     0.00000          NA          NA          NA            NA
b_wifi          0.95151    0.052893      17.989    0.055165        17.248
b_food          0.41168    0.052141       7.895    0.052807         7.796
```

```
> apollo_deltaMethod(model,
+                    deltaMethod_settings = list(
+                      expression=c(VTT="b_tt/b_cost",
+                                   VAT="b_access/b_cost",
+                                   VWIFI="(b_wifi-b_no_frills)/b_cost",
+                                   VFOOD="(b_food-b_no_frills)/b_cost")))
The expression VWIFI includes parameters that were fixed in estimation: b_no_frills
These have been replaced by their fixed values, giving: (b_wifi-0)/b_cost

The expression VFOOD includes parameters that were fixed in estimation: b_no_frills
These have been replaced by their fixed values, giving: (b_food-0)/b_cost

Running Delta method computation for user-defined function:

 Expression    Value Robust s.e. Rob t-ratio (0)
        VTT   0.2053      0.0095           21.56
        VAT   0.3393      0.0423            8.03
      VWIFI -16.2084      1.0033          -16.15
      VFOOD  -7.0127      0.8949           -7.84
```

```
Estimated parameters                        : 8
Time taken (hh:mm:ss)                       :  00:00:1.32
     pre-estimation                         :  00:00:0.62
     estimation                             :  00:00:0.21
          initial estimation                :  00:00:0.16
          estimation after rescaling        :  00:00:0.04
     post-estimation                        :  00:00:0.5
Iterations                                  :  11
     initial estimation                     :  10
     estimation after rescaling             :  1

Unconstrained optimisation.

Estimates:
               Estimate        s.e.   t.rat.(0)    Rob.s.e. Rob.t.rat.(0)
asc_car         0.00000          NA          NA          NA            NA
asc_bus        -2.04288    0.075132     -27.191    0.092220       -22.152
asc_air        -0.58780    0.180223      -3.262    0.197274        -2.980
asc_rail       -0.86198    0.107216      -8.040    0.117824        -7.316
v_tt            0.20533    0.008783      23.379    0.009523        21.563
v_access        0.33933    0.042442       7.995    0.042270         8.028
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```

Incorporating deterministic heterogeneity

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Incorporating deterministic heterogeneity

Care is required

- Using VTT as an example
- Utility space
  - include heterogeneity in cost and/or time sensitivity
  - either of these will lead to heterogeneity in VTT
- WTP space
  - still matters where we include heterogeneity
  - explicit calculation of VTT may be required

Incorporating deterministic heterogeneity

Example 1: heterogeneity in time sensitivity

- Let C_{jn} and T_{jn} give cost and time attributes, respectively
- Utility space

$$V_{jn} = \beta_C \cdot C_{jn} + (\beta_T + \beta_{T,male} \cdot z_{male,n}) \cdot T_{jn}$$

$$VTT_n = \frac{\partial V_{jn}}{\partial T_{jn}} / \frac{\partial V_{jn}}{\partial C_{jn}} = \frac{\beta_T + \beta_{T,male} \cdot z_{male,n}}{\beta_C}$$

- WTP space

$$V_{jn} = \beta_C \cdot [C_{jn} + (\beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n}) \cdot T_{jn}]$$

$$VTT_n = \frac{\partial V_{jn}}{\partial T_{jn}} / \frac{\partial V_{jn}}{\partial C_{jn}} = \frac{\beta_C \cdot [\beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n}]}{\beta_C} = \beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n}$$

  - no need to consider $\frac{\partial V_{jn}}{\partial C_{jn}}$ in this case
  - VTT still depends on gender

Incorporating deterministic heterogeneity

Example 2: heterogeneity in time and cost sensitivity

- Let inc_n be the income of person n
- Utility space

$$V_{jn} = \beta_C \cdot C_{jn} \cdot \left(\frac{inc_n}{\overline{inc_n}}\right)^{\lambda_{inc}} + \left(\beta_T + \beta_{T,male} \cdot z_{male,n}\right) \cdot T_{jn}$$

$$VTT_n = \frac{\partial V_{jn}}{\partial T_{jn}} / \frac{\partial V_{jn}}{\partial C_{jn}} = \frac{\beta_T + \beta_{T,male} \cdot z_{male,n}}{\beta_C \cdot \left(\frac{inc_n}{\overline{inc_n}}\right)^{\lambda_{inc}}}$$

- VTT depends on gender and income

Incorporating deterministic heterogeneity

Example 2: correct WTP space implementation

- Income needs to be interacted with cost

$$V_{jn} = \beta_C \cdot \left[C_{jn} \cdot \left(\frac{inc_n}{\overline{inc_n}} \right)^{\lambda_{inc}} + (\beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n}) \cdot T_{jn} \right]$$

$$VTT_n = \frac{\partial V_{jn}}{\partial T_{jn}} \Big/ \frac{\partial V_{jn}}{\partial C_{jn}} = \frac{\beta_C \cdot (\beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n})}{\beta_C \cdot \left(\frac{inc_n}{\overline{inc_n}} \right)^{\lambda_{inc}}} = \frac{\beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n}}{\left(\frac{inc_n}{\overline{inc_n}} \right)^{\lambda_{inc}}}$$

- VTT depends on gender and income
- but need to consider $\frac{\partial V_{jn}}{\partial C_{jn}}$ in this case!

Incorporating deterministic heterogeneity

Example 2: incorrect WTP space implementation

$$V_{jn} = \beta_C \cdot \left(\frac{inc_n}{\overline{inc_n}}\right)^{\lambda_{inc}} \cdot \left[C_{jn} + (\beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n}) \cdot T_{jn}\right]$$

$$VTT_n = \frac{\partial V_{jn}}{\partial T_{jn}} \Big/ \frac{\partial V_{jn}}{\partial C_{jn}} = \frac{\beta_C \cdot \left(\frac{inc_n}{\overline{inc_n}}\right)^{\lambda_{inc}} \cdot (\beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n})}{\beta_C \cdot \left(\frac{inc_n}{\overline{inc_n}}\right)^{\lambda_{inc}}} = \beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n}$$

- Income now only affects scale of utility
- Cancels out in VTT calculation
- Not what we want!

Incorporating deterministic heterogeneity

Example 2: another *incorrect* WTP space implementation

$$V_{jn} = \beta_C \cdot \left[C_{jn} + (\beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n}) \cdot T_{jn} \cdot \left(\frac{inc_n}{\overline{inc_n}} \right)^{\lambda_{inc}} \right]$$

$$VTT_n = \frac{\partial V_{jn}}{\partial T_{jn}} \Big/ \frac{\partial V_{jn}}{\partial C_{jn}} = \frac{\beta_C \cdot (\beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n}) \cdot \left(\frac{inc_n}{\overline{inc_n}} \right)^{\lambda_{inc}}}{\beta_C}$$

$$= (\beta_{VTT} + \beta_{VTT,male} \cdot z_{male,n}) \cdot \left(\frac{inc_n}{\overline{inc_n}} \right)^{\lambda_{inc}}$$

- Income now affects VTT, but through impact on time, not cost
- No longer the same as utility space specification we had

Incorporating random heterogeneity

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Experimental design

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Incorporating random heterogeneity

Specification

- Utility space: $V_i = \sum_k \beta'_k x_{ik}$
- WTP space: $V_i = \beta_c \left(c_{ik} + \sum_{k \neq c} \beta'_{valuation,k} x_{ik} \right)$
- With non-random coefficients, these are equivalent
- No longer necessarily the case with random coefficients
  - distributional assumptions change
- WTP space with mixture models
  - avoids need to divide by random β_c
  - should still not mean that we use Normal distribution!

Incorporating random heterogeneity

Illustration for Mixed Logit (Lognormal)

- Negative LN for cost coefficient
- Positive LN for valuations
- Fit worse than preference space
  - $-1,457.14$ *vs* $-1,445.69$
- VTT is lower too
  - mean: 22.34 *vs* 40.19
  - sd: 16.46 *vs* 57.69
- Not because one model is better than the other
- Simply a result of different distributional assumptions
  - WTP space implies positive correlation

```
Estimates:
                 Estimate        s.e.   t.rat.(0)    Rob.s.e. Rob.t.rat.(0)
mu_log_v_tt      -1.20493     0.02149    -56.0770    0.013989      -86.1365
sigma_log_v_tt    0.65920     0.01585     41.5791    0.007371       89.4259
mu_log_b_tc      -0.04223     0.15957     -0.2647    0.163462       -0.2584
sigma_log_b_tc   -1.59010     0.18695     -8.5054    0.164220       -9.6827
mu_log_v_hw      -2.48815     0.06321    -39.3658    0.049127      -50.6474
sigma_log_v_hw   -1.14667     0.04307    -26.6229    0.031392      -36.5279
mu_log_v_ch       1.28881     0.04251     30.3201    0.026470       48.6886
sigma_log_v_ch   -1.16747     0.04289    -27.2181    0.025292      -46.1600

> unconditionals = apollo_unconditionals(model,apollo_probabilities,apollo_inputs)
Unconditional distributions computed
> mean(60*as.vector(unconditionals[["v_tt"]]))
[1] 22.34339
> sd(60*as.vector(unconditionals[["v_tt"]]))
[1] 16.45696
```

Observations

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Observations

WTP space: what is the benefit?

- With simple models, of course straightforward to calculate WTP as ratios of partial derivatives
- Calculation of standard errors using Delta method is not too hard either
- Avoids the need for a non-linear utility specification
- Benefits of WTP space arise specifically in the context of models with random heterogeneity, where ratio of marginal utilities may not be well defined

Observations

WTP space: common confusions

Confusion 1	Papers describe WTP space is a different model
Truth	WTP space is not a model, but a different parameterisation of utility
Confusion 2	Papers state that WTP space fits better than utility space, or vice versa
Truth	This only happens as a result of different distributional assumptions in mixed logit, not because one specification is superior to another

Observations

WTP space: common confusions

Confusion 3 Some people say cost coefficient in WTP space is constrained to 1

Truth Confusion arises from $V_i = \mu\left(\delta_{wtp,i} + \sum_k^K \beta_{wtp,k} x_{k,i} + cost_i\right)$, where k are all non-cost attributes. But $\mu = \beta_{cost}$.

Confusion 4 All parameters and ASC need to be included in scaling by β_{cost}

Truth No, up to the user which parameters to express in monetary terms, and these 4 specifications are all equivalent, and final 3 are all WTP space

$$V_i = \delta_i + \beta_1 x_{1,i} + \beta_2 x_{2,i} + \beta_{cost} cost_i$$

$$V_i = \beta_{cost}\left(\delta_{wtp,i} + \beta_{wtp,1} x_{1,i} + \beta_{wtp,2} x_{2,i} + cost_i\right)$$

$$V_i = \delta_i + \beta_{cost}\left(\beta_{wtp,1} x_{1,i} + \beta_{wtp,2} x_{2,i} + cost_i\right)$$

$$V_i = \delta_i + \beta_2 x_{2,i} + \beta_{cost}\left(\beta_{wtp,1} x_{1,i} + cost_i\right)$$

Observations

WTP space: common confusions

Confusion 5 Non-cost attributes should have a negative sign in front of them

Truth A user may find this convenient, but it is not a requirement - only implies that WTP is for increases in attribute (remember the earlier point)

$$V_i = \beta_{cost}\left(\delta_{wtp,i} - \beta_{wtp,1}x_{1,i} - \beta_{wtp,2}x_{2,i} + cost_i\right)$$

Confusion 6 Only cost can be used for scaling

Truth A model can be parameterised in any other valuation space, using whatever attribute the user would use as the denominator in MRS